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# Induced explode-decay dromions in the $(2+1)$ dimensional nonisospectral nonlinear Schrödinger (NLS) equation

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**Abstract.** In this paper, we have identified a novel method of inducing localized solutions in the (2+1) dimensional nonisospectral nonlinear Schrödinger (NLS) equation by utilising the freedom in the system. The new class of localized solutions includes induced dromions, lump dromions, dipole dromions etc. The amplitude of the localized solutions generated is found to explode or decay with time. We have also brought out the interaction of induced dromions.

**PACS.** 02.30.Jr Partial differential equations – 04.30.Nk Wave propagation and interactions – 05.45.Yv Solitons

## 1 Introduction

The advent of dromions [1–6] which has given a new perspective to the concept of integrability in (2+1) dimensional nonlinear dynamical systems and the flurry of activities which followed this discovery have kept the interests alive in the investigation of higher dimensional nonlinear systems. Such exponentially localized solutions arise by feeding energy from the boundaries or lower dimensional arbitrary functions of space and time present in the system. The presence of such lower dimensional arbitrary functions of space and time have enriched the structure of solutions leading to the construction of more exotic structures in the (2+1) dimensional nonlinear partial differential equations (pdes) [7]. However, it should be mentioned that the integrable equations with constant coefficients are regarded to be highly idealized in physical situations. Hence, it is believed that equations with variable coefficients and nonisospectral eigen parameters may provide more realistic models and would enable us to get a better understanding of the physical phenomena around us. This led to an upsurge in the investigation of inhomogeneous integrable models [8–16]. Hence, one may ask do such exotic structures exist in such systems as well? and if so, how they can be generated? What are their characteristics? The answer to the above questions will open up the possibility of constructing such exotic structures in (2+1) dimensional nonisospectral nonlinear pdes.

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## 2 $(2+1)$ dimensional nonisospectral NLS equation and oscillating line solitons

In this letter, we consider the following nonisospectral (2+1) dimensional nonlinear Schrödinger (NLS) equation of the form [17]

$$i\psi_t - \psi_{xy} - i(\rho\psi)_x - R\psi = 0 \tag{1a}$$
$$R_x = (1/2)\partial_y|\psi|^2 \tag{1b}$$

where $\psi = \psi(x, y, t)$ is a complex field variable and $R = R(x, y, t)$ is a real potential. The function $\rho(x)$ varies linearly with $x$ of the form

$$\rho(x) = \mu_3 x + \nu_3 \tag{2}$$

where $\mu_3$ and $\nu_3$ are constants. When $\rho(x) = 0$, equation (1) reduces to the isospectral (2+1) dimensional NLS equation

$$i\psi_t = \psi_{xy} + R\psi \tag{3a}$$
$$R_x = (1/2)\partial_y|\psi|^2. \tag{3b}$$

This equation has been investigated earlier and induced localized solutions were unearthed by judiciously harnessing the arbitrary functions in the system [18,19]. Eventhough the Painleve property of equation (1) has not been established yet, it is known to be completely integrable and admits the Lax-pair [17].

Equation (1) under the following dependent variable transformation,

$$\psi = G/F \tag{4a}$$

$$R = 2\partial_{xy} \log F \tag{4b}$$

can be transformed into the Hirota from

$$(iD_t - D_x D_y - i\mu_3 - i\rho D_x) G \cdot F = 0 \tag{5a}$$

$$2D_x^2 F \cdot F = GG^*. \tag{5b}$$

We now expand $G$ and $F$ in the form of a power series as

$$G = \epsilon g^{(1)} + \epsilon^3 g^{(3)} + \cdots \tag{6a}$$

$$F = 1 + \epsilon^2 f^{(2)} + \epsilon^4 f^{(4)} + \cdots \tag{6b}$$

To generate line solitons, we now substitute (6) into (5) and collect the resultant equations by comparing various powers of $\epsilon$ to obtain

$$\epsilon:\quad ig_t^{(1)} - g_{xy}^{(1)} - i\mu_3 g^{(1)} - i\rho g_x^{(1)} = 0 \tag{7a}$$

$$\epsilon^2:\quad 4f_{xx}^{(2)} = g^{(1)} g^{(1)*} \tag{7b}$$

$$\epsilon^3:\quad ig_t^{(3)} - g_{xy}^{(3)} - i\mu_3 g^{(3)} - i\rho g_x^{(3)} = -(iD_t - D_x D_y - i\mu_3 - i\rho D_x) g^{(1)} f^{(2)} \tag{7c}$$

$$\epsilon^4:\quad 2D_x^2 (2f^{(4)} + f^{(2)} \cdot f^{(2)}) = g^{(1)} g^{(3)*} + g^{(3)} g^{(1)*} \tag{7d}$$

and so on. Equation (7a) can be solved to obtain

$$g^{(1)} = \sum_{j=1}^{N} \exp(\chi_j),$$

$$\chi_j = k_j(t) x + l_j y + \int \Omega_j(t) dt + c_j \tag{8a}$$

$$\Omega_j(t) = -il_j k_j(t) + \nu_3 k_j(t) + \mu_3 \tag{8b}$$

$$(k_j)_t = \mu_3 k_j \tag{8c}$$

where $l_j$ is a complex parameter while $k_j(t)$ and $\Omega_j(t)$ are complex functions of time and they are related by the dispersion relation (8b). To construct one soliton solution, we take $N = 1$ and substitute $g^{(1)}$ in (7b) to give

$$f^{(2)} = e^{\chi_1 + \chi_1^* + 2\phi}, \quad e^{2\phi} = \frac{1}{16k_{1R}^2(t)}. \tag{9}$$

Using $g^{(1)}$ and $f^{(2)}$ in (7c) and (7d), one can prove that $g^{(j)} = 0$ for $j \geq 3$ and $f^{(j)} = 0$ for $j \geq 4$. Now, using equations (4a) and (4b), the physical field $\psi$ and the potential $R$ can be expressed as

$$\psi = 2k_{1R}(t) \mathrm{sech}(\chi_{1R} + \phi) \exp(i\chi_{1\mathrm{I}}) \tag{10a}$$

$$R = 2l_{1R} k_{1R}(t) \mathrm{sech}^2(\chi_{1R} + \phi). \tag{10b}$$

Looking at the above solutions, we observe that both the physical field and the potential are driven by the line solitons as they remain finite on the line

$$C_1 = \chi_{1R} + \phi = k_{1R}(t) x + l_{1R} y + \int \Omega_{1R}(t) dt + c_{1R} = 0 \tag{11}$$

and their amplitudes oscillate with time as it is evident from equations (10a, 10b). A snapshot of $|\psi|$ and $R$ is shown in Figures 1 and 2.

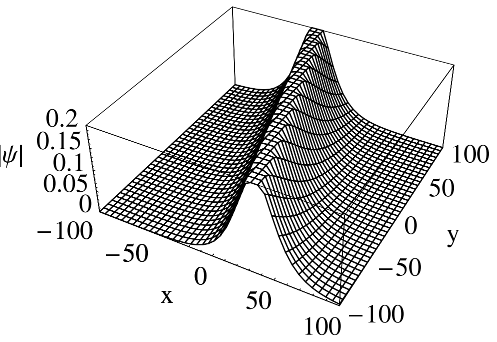


**Fig. 1.** Line soliton for the physical field $\psi$ at $t = -10$.

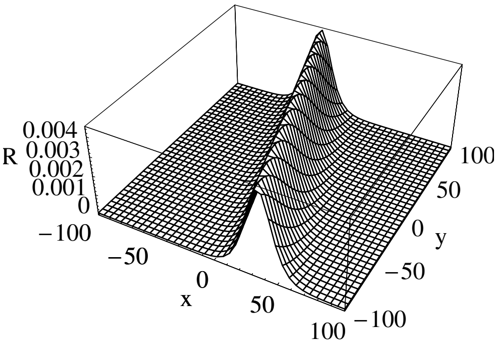


**Fig. 2.** Line soliton for the potential $R$ at $t = -10$.

## 3 Explode decay induced dromions

It should be mentioned that the system (1) does not admit ghost solitons (one dimensional solitons driving potentials in the absence of physical field) and hence it does not possess basic dromions. However, one can induce localized solutions in the system by properly harnessing the arbitrary functions of space and time. Looking at equation (1b), one finds that there exists a lower dimensional arbitrary function of space and time $h_R(y,t)$ of the form

$$R = (1/2) \int (|\psi|^2)_y dx + h_R(y,t). \tag{12}$$

Introducing the above arbitrary function in the solution of equation (7a), we have

$$g^{(1)} = \sum_{j=1}^{N} \exp(\chi_\mathrm{j}), \quad \chi_j = k_j(t) x + \int \Omega_j(t) dt + h_j(\rho) + c_j \tag{13}$$

where $h_j(\rho)$ is a complex function of space and time of the form

$$h_j(\rho) = h_j(y - ik_j t) \tag{14a}$$

and

$$\Omega_j = \mu_3 + \nu_3 k_j. \tag{14b}$$

Thus, the physical field $\psi$ and the potential $R$ can now be expressed as

$$\psi = 2k_{1R}(t) \mathrm{sech}(\chi_{1R} + \phi) \exp(i\chi_{1\mathrm{I}}) \tag{15}$$

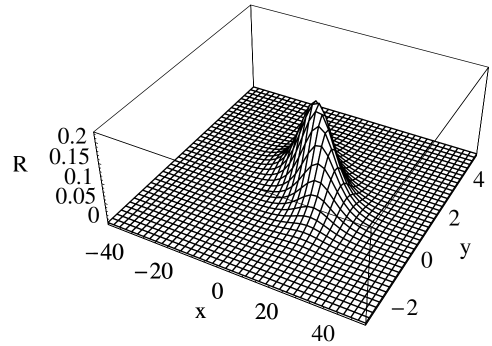


**Fig. 3.** A snapshot of the induced dromion.

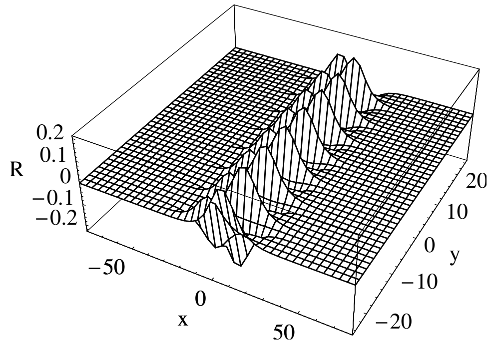


**Fig. 5.** Dipole dromion at $t = -10$.

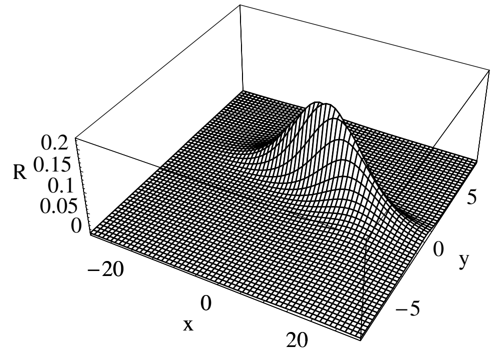


**Fig. 4.** Lump dromion at $t = -10$.

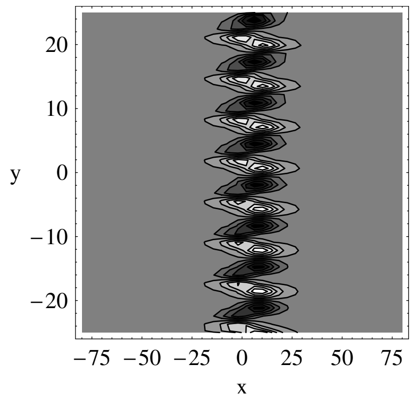


**Fig. 6.** Contour plot of dipole dromion.

$$R = 2k_{1R}(t)(h_{1R})_{\rho_R}\text{sech}^2(\chi_{1R}+\phi), \quad \rho_R = y+k_{1I}(0)e^{\mu_3 t}t. \tag{16}$$

Now, we find that $\psi$ and R are now driven by curved solitons as they remain finite on the curve $C_2 = k_{1R}(t) + \int \Omega_{1R}(t)dt + h_{1R}(\rho_R) + c_2 + \phi = 0$ with their amplitudes again varying with time. To generate a single dromion for the potential $R$, we now harness the arbitrary function $h_{1R}$ as

$$h_{1R}(\rho_R) = \tanh(\rho_R) \tag{17}$$

so that the one dromion solution assumes the following form

$$R = 2k_{1R}(t)\text{sech}^2(\rho_R)\text{sech}^2(\chi_{1R}+\phi). \tag{18}$$

From the above equation, it is clear that the one dromion originates by virtue of the interaction of the line soliton $\text{sech}^2(\rho_R)$ with the curved soliton $\text{sech}^2(\chi_{1R}+\phi)$ and decays exponentially in all directions as shown in Figure 3. It should be mentioned that this is the first time such localized solutions have been induced in a nonisospectral (2+1) dimensional nonlinear partial differential equation. It is also obvious that the amplitude of the above exponentially localized solution explodes (or decays) with time and hence one calls such solutions as "explode decay induced dromions". One can as well replace $\rho_R$ by the arbitrary function $h_{1R}(\rho_R)$ so that one can obtain an exponentially localized solution driven by two curved solitons. One can construct even more richer structures for the potential by properly harnessing the arbitrary function $h_{1R}(\rho_R)$.

For example, the choice of the arbitrary function

$$h_{1R}(\rho_R) = \text{Arc}\tan(\rho_R+\rho_0) \tag{19}$$

generates a lump dromion of the form (shown in Fig. 4)

$$R = 2k_{1R}(t)\frac{\text{sech}^2(\chi_{1R}+\phi)}{1+(\rho_R+\rho_0)^2} \tag{20}$$

while choosing

$$h_{1R}(\rho_R) = \int cn(\rho_R)d(\rho_R) \tag{21}$$

generates a dipole dromion (shown in Fig. 5) as

$$R = 2k_{1R}(t)cn(\rho_R)\text{sech}(\chi_{1R}+\phi). \tag{22}$$

The contour plot of the dipole dromion (displayed in Fig. 6) shows that the amplitude of $R$ is both positive as well as negative. Again, all these solutions either explode or decay with time.

It should be mentioned that as the system does not support basic dromions due to the absence of ghost solitons, it does not possess basic multidromions. However, one can induce multi induced dromions again by suitably mobilizing the arbitrary function $h_{1R}(\rho_R)$. For example, a two induced dromion can be generated by the interaction of two one dimensional solitons (one decaying algebraically and the other exponentially) moving with different velocities with the curved soliton $\text{sech}^2(\chi_{1R}+\phi)$ and has the

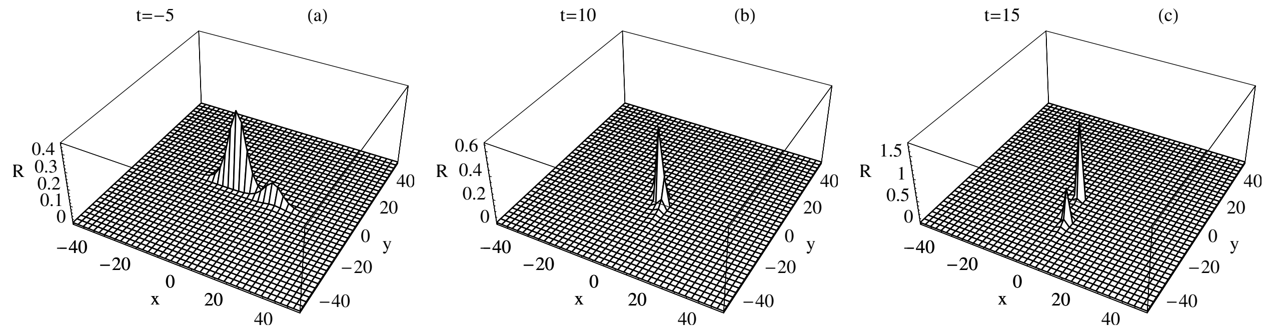


**Fig. 7.** Induced dromion interaction.

following form

$$R = 2k_{(1R)}(t)\left[\frac{2}{(\rho_{1R}+\rho_0)^2+1}+\mathrm{sech}^2(\rho_{2R})\right]\mathrm{sech}^2(\chi_{1R}+\phi)$$

$$\rho_{1R} = y + k_{1I}(0)e^{\mu_3 t}t, \quad \rho_{2R} = y + k_{2I}(0)e^{\mu_3 t}t. \quad (23)$$

Figure 7 shows the collision of two induced dromions. Looking at Figures 7a–7c, we observe that they undergo inelastic collision and gain amplitude after interaction.

## 4 Discussion

In this article, we have investigated the (2+1) dimensional nonisospectral NLS equation and suitably mobilized the lower dimensional arbitrary function of space and time to induce localized solutions in the system. The question of integrability of the system by virtue of the introduction of time in the function $\rho(x)$ and also for a more general $\rho$ (other than a linear function of $x$) and the existence of localized solutions remain to be explored.

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